The Statistical Validity of Using Ratio Variables in Human Kinetics Research

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Introduction

- A ratio variable is a composite variable which consists of a numerator variable and a denominator variable
- Ratio variables are used:
  - to standardize data with the deflation of one variable by another
  - as a measurement variable of direct interest, without any intent to derive a denominator-free variable

Introduction

- Ratio variables are commonly used variables in human kinetics research
  - 62% of the articles published in RQES (1992-95) used ANOVA or MANOVA, and 39% of them used ratio variables

Introduction

- Deflation issues
  - The impact of the denominator on the validity of using ratio variable needs to be investigated
  - Simple ratio variables are used without any validation

Introduction

- Reliability issues
  - Information on the reliability of commonly used ratio variables is not available
- Issues in RM ANOVA
  - We do not know to what extent the ratio variable affects the circularity of the matrix of ratio scores
  - It is not clear how a ratio affects the sampling characteristics of the variance and covariance matrix, thus the ratio effect on the Type I error rate and power needs to be investigated

Purposes of the Study

- Evaluate the validity of the simple ratio and alternate deflation models
- Examine the reliability of some commonly used ratio variables
- Investigate the effect of using ratio variables in RM ANOVA
  - How does the ratio variable affect the characteristics of the population Epsilon?
  - What are the sampling characteristics of the Epsilon estimate?
  - How do ratio variables affect type I error rates and power?

Methods for Evaluation of Ratio Variables

- Variable selection
- Deflation model evaluation
- Reliability evaluation

Evaluation of Ratio Variables

- Variable selection
  - Four of the most commonly used ratio variables in human kinetics were selected
    - » Vo2max/kg, n=52, (Taunton, 1992)
    - » DLco/VA, n=13, (Bacon, 1997)
      - diffusing capacity of the lungs for carbon monoxide (DLco) divided by alveolar ventilation volume
    - » Nondominant/Dominant forearm strength, n=43, (Kramer, 1994)
    - » Waist/Hip (girth), n=1851, (Sonnenschein, 1993)

Evaluation of Ratio Variables (cont.)

- **Deflation models**
  - Simple ratio:
    - » $Y=X_1/X_2$
  - Linear regression model:
    - » $Y=(X_1-a)/X_2$ (LRM)
  - Nonlinear regression models:
    - » $Y=X_1/X_2^k$ (NLRM1)
    - » $Y=(X_1-a)/X_2^k$ (NLRM2)

Evaluation of Ratio Variables (cont.)

- **Deflation model evaluation**
  - The simple ratio and the adjusted ratios were evaluated by using the following criteria:
    - » Statistical criterion: Pearson $r_{y/x2} = 0$
    - » Graphical criterion: No distortion of the variance of Y when the denominator increases
    - » Algebraic criterion: $\hat{\beta}_t = \bar{Y}_t$
    - » R^2: High R^2
    - » Reliability: High reliability
- **Reliability evaluation**
  - Intraclass correlation

Results and Discussion
- Evaluation of Ratio Variables

- Ratio variables used for deflation
- Ratio variables used as measurement variables

Evaluation of Deflation Models

- **Data Set 1** (Taunton, 1992)**:**
  - Variable: Vo2max/body mass
  - Subjects: Healthy women aged 65 to 75, n=52
  - Measurement: Three time points during twelve weeks

Simple Ratio Evaluation

Ratio Model	r X_1 vs X_2	r Y vs X_2	Estimated Expected Value ^β	Empirical Mean $\bar{Y}$
$Y=X_1/X_2$				
Trial 1	0.48	-0.45	18.28	18.85
Trial 2	0.54	-0.61	19.57	20.22
Trial 3	0.46	-0.58	20.30	21.02

	Reliability Coefficient		
	X1	X_2	Y
Intraclass r			
Average Trials	0.88	0.99	0.89
Single Trial	0.72	0.99	0.72

Alternative Model Evaluation

- **There is no substantial difference among the alternative models for these data**
  - The alternative ratio variables have a near-zero correlation with the denominator
  - The denominator (body mass) related distortion of variance was not shown
  - The expected values were accurately estimated by all three models
  - The values of R^2 were slightly different among the three models
  - There was no substantial difference in the reliability among the alternative ratios for these data
- **The LRM and NLRM1 may be more preferable than NLRM2 for these data**

A Summary of the Evaluation of the Deflation Models

- One should not assume that a simple ratio model is valid for deflation purposes
- A optimal deflation model useful for all variables may not exist. Different deflation models should be used to fit each empirical data set for determining the best deflation model
- For developing the best deflation model, the criteria developed in this study could be used

Ratio Variables Used as direct Interests

- **Nondominant/Dominant Ratio (Pronation)**

Pronation Torques	Nondominant T1	Nondominant T2	Dominant T1	Dominant T2	Nondominant/Dominant T1	Nondominant/Dominant T2
Mean	8.01	8.24	8.95	8.98	0.91	0.94
Sd	3.71	3.58	4.12	3.97	0.22	0.16
V	0.46	0.43	0.46	0.44	0.24	0.17
Reliability All Trials	0.97		0.94		0.68	
Single Trial	0.94		0.89		0.51	

Ratio Variables Used as direct Interests (continue)

- **Waist/Hip Ratio**

	Waist			Hip			W/H Ratio		
	T1	T2	T3	T1	T2	T3	T1	T2	T3
Mean	74.93	76.27	76.74	97.71	100.83	100.96	0.77	0.76	0.76
Sd	11.64	11.97	11.80	10.77	10.00	9.86	0.07	0.07	0.07
V	0.16	0.16	0.15	0.11	0.10	0.10	0.09	0.10	0.10
Reliability									
All Trials		0.96			0.94			0.89	
Single Trial		0.89			0.84			0.73	

A Discussion on the Reliability of the Ratio Variable: Data sets 3 and 4

- If the coefficients of variation (V) of the component variables are the same the reliability of the ratio variable is not affected by the coefficients of variation of the components
- When the coefficients of variation of the component variables are different, the reliability of the ratio variable is higher than when the coefficients of variation are equal
- Although the components are highly reliable, the derived ratio variable data may have low reliability
- Whenever a ratio variable is used, the reliability of the derived ratio variable data should be computed and evaluated

Methods for Computer Simulation Investigation

- Overview of the simulation conditions
- The procedures
  - Data generation
  - Data analysis

Overview of simulation conditions

Simulation Conditions

- **Population Epsilon**
  - Epsilon(x1)=1.0, Epsilon(x2)=1.0
  - Epsilon(x1)=.70, Epsilon(x2)=.70
  - Epsilon(x1)=.70, Epsilon(x1)=1.0
  - Epsilon(x1)=1.0, Epsilon(x1)=.70
- **Population Means Over Trials**
  - μ_{x1}=75
  - μ_{x2}=100
- **Correlation ρ_{x1x2}**
  - .90
  - .70
  - .50
- **Variation Coefficients**
  - Vx1/Vx2=2.0
  - Vx1/Vx2 =1.0
  - Vx1/Vx2 =0.5

Preparation of covariance matrices

	X1(numerator)					*X2(Denominator)*				
X1	*Trial 1*	*Trial 2*	*Trial 3*	*Trial4*	*Trial5*	*Trial 1*	*Trial 2*	*Trial 3*	*Trial4*	*Trial5*
Trial 1	$Var(X_1)_1$	$Cov(X_1)_{12}$	$Cov(X_1)_{13}$	$Cov(X_1)_{14}$	$Cov(X_1)_{15}$	$Cov(X_{11}X_{21})$	$Cov(X_{11}X_{22})$	$Cov(X_{11}X_{23})$	$Cov(X_{11}X_{24})$	$Cov(X_{11}X_{25})$
Trial 2		$Var(X_1)_2$	$Cov(X_1)_{23}$	$Cov(X_1)_{24}$	$Cov(X_1)_{25}$	$Cov(X_{12}X_{21})$	$Cov(X_{12}X_{22})$	$Cov(X_{12}X_{23})$	$Cov(X_{12}X_{24})$	$Cov(X_{12}X_{25})$
Trial 3			$Var(X_1)_3$	$Cov(X_1)_{34}$	$Cov(X_1)_{35}$	$Cov(X_{13}X_{21})$	$Cov(X_{13}X_{22})$	$Cov(X_{13}X_{23})$	$Cov(X_{13}X_{24})$	$Cov(X_{13}X_{25})$
Trial 4				$Var(X_1)_4$	$Cov(X_1)_{45}$	$Cov(X_{14}X_{21})$	$Cov(X_{14}X_{22})$	$Cov(X_{14}X_{23})$	$Cov(X_{14}X_{24})$	$Cov(X_{14}X_{25})$
Trial 5					$Var(X_1)_5$	$Cov(X_{15}X_{21})$	$Cov(X_{15}X_{22})$	$Cov(X_{15}X_{23})$	$Cov(X_{15}X_{24})$	$Cov(X_{15}X_{25})$

X2	*Trial1*	*Trial2*	*Trial3*	*Trial4*	*Trial5*	*Trial1*	*Trial2*	*Trial3*	*Trial4*	*Trial5*
Trial 1						$Var(X_2)_1$	$Cov(X_2)_{12}$	$Cov(X_2)_{13}$	$Cov(X_2)_{14}$	$Cov(X_2)_{15}$
Trial 2							$Var(X_2)_2$	$Cov(X_2)_{23}$	$Cov(X_2)_{24}$	$Cov(X_2)_{25}$
Trial 3								$Var(X_2)_3$	$Cov(X_2)_{34}$	$Cov(X_2)_{35}$
Trial 4									$Var(X_2)_4$	$Cov(X_2)_{45}$
Trial 5										$Var(X_2)_5$

Data Generation and Analysis

- A FORTRAN program was developed to conduct the data generation and analysis
- The multivariate random number generator in IMSL was called to generate raw score populations (k=5, N=90,000)
- Ratio population data were derived by transformation

Results and Discussion
- Computer Simulation Investigation

- The effect of using ratios on ε, type I error
  - Population ε
  - Correlation of ratio variables
  - Sampling characteristics of ^ε
  - Type I error
- The effect of using ratios on power

Values of Epsilon for X_1/X_2 Under Varying Conditions of X_1 and X_2

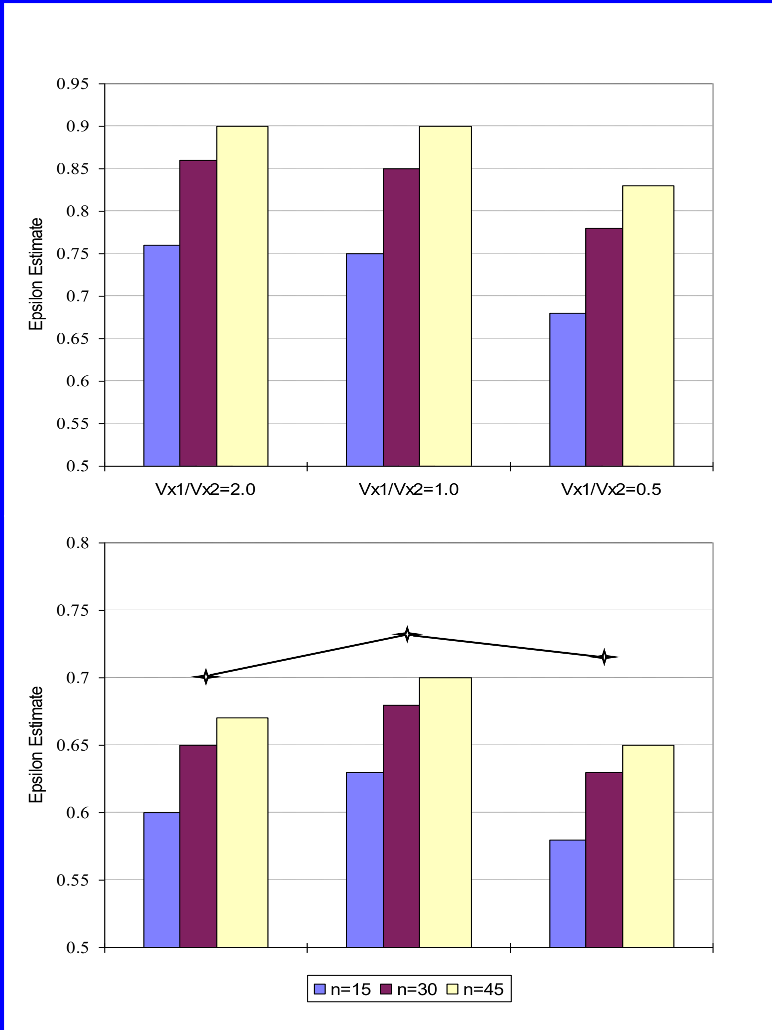
	$V_{x1}/V_{x2}=2.0$	$V_{x1}/V_{x2}=1.0$	$V_{x1}/V_{x2}=0.5$
$\varepsilon_{X1}=\varepsilon_{X2}=1.0$			
$\rho_{X1X2}=0.9$	1.00	1.00	1.00
$\rho_{X1X2}=0.7$	1.00	1.00	1.00
$\rho_{X1X2}=0.5$	1.00	1.00	1.00
$\varepsilon_{X1}=\varepsilon_{X2}=0.7$			
$\rho_{X1X2}=0.9$	0.70	0.73	0.72
$\rho_{X1X2}=0.7$	0.70	0.71	0.72
$\rho_{X1X2}=0.5$	0.70	0.71	0.72
$\varepsilon_{X1}=0.7$ $\varepsilon_{X2}=1.0$			
$\rho_{X1X2}=0.9$	0.59	0.87	0.86
$\rho_{X1X2}=0.7$	0.64	0.91	0.96
$\rho_{X1X2}=0.5$	0.70	0.91	0.99
$\varepsilon_{X1}=1.0$ $\varepsilon_{X2}=0.7$			
$\rho_{X1X2}=0.7$	0.78	0.81	0.71
$\rho_{X1X2}=0.5$	0.99	0.84	0.72

Inter-trial Correlations of Ratio Variables

- $\varepsilon_{x1}=\varepsilon_{x2}=1.0$
  - ρ_{yiyj} show a constant correlation pattern
- $\varepsilon_{x1}=\varepsilon_{x2}=0.7$
  - ρ_{yiyj} exhibit a decreasing pattern but not the simplex pattern
- $\varepsilon_{x1}=0.7$, $\varepsilon_{x2}=1.0$ and $\varepsilon_{x1}=1.0$, $\varepsilon_{x2}=0.7$
  - ρ_{yiyj} pattern is mostly affected by the component variable which has the larger coefficient of variation

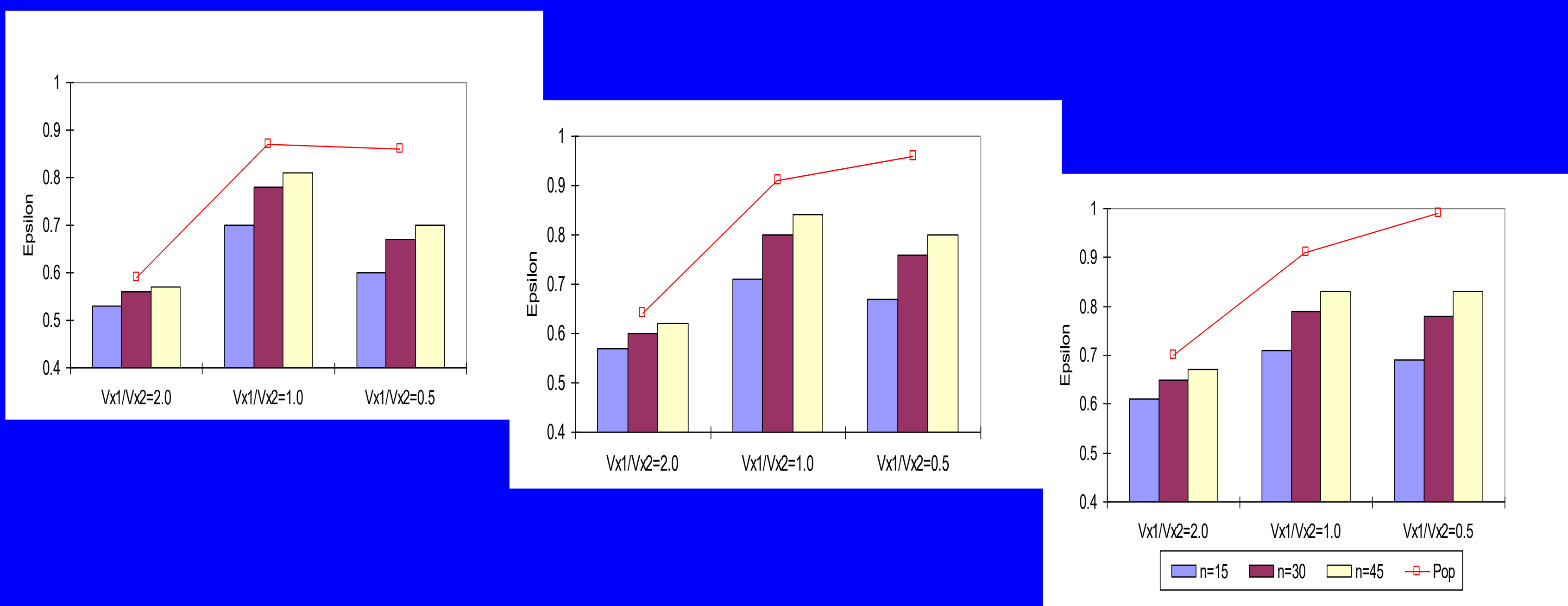
Sampling Characteristics of the ε Estimate

- $\varepsilon_{x1}=\varepsilon_{x2}=1.0$ and $\varepsilon_{x1}=\varepsilon_{x2}=0.7$
  - $\hat{\varepsilon}_{x1/x2}$ is a downward biased estimator and virtually unaffected by ρ_{x1x2}
  - When V_{x1} increases the bias of $\hat{\varepsilon}_{x1/x2}$ increases
  - V_{x1}/V_{x2} has less effect in the condition $\varepsilon_{x1}=\varepsilon_{x2}=0.7$



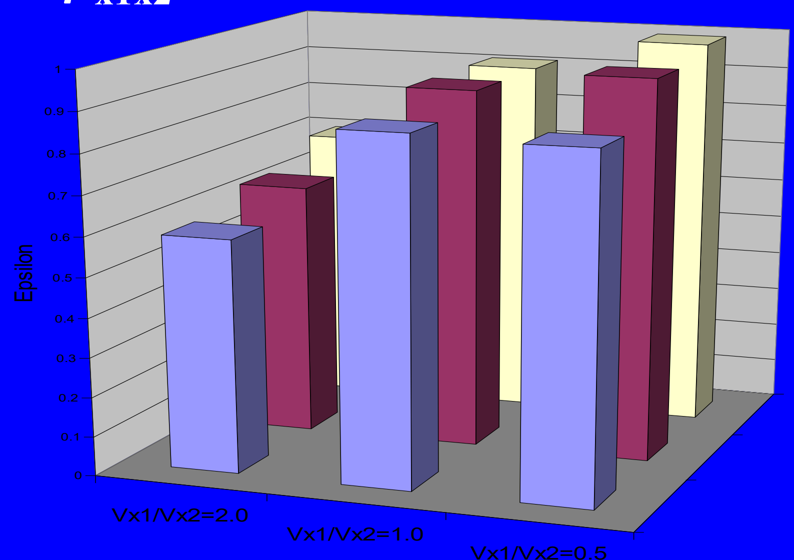
Sampling Characteristics of the ε Estimate
(continue)

- ε_{x1}=0.7 and ε_{x2}=1.0
  - Both V_{x1}/ V_{x2} and ρ_{x1x2} affect the bias of $\hat{\varepsilon}_{x1/x2}$
  - When V_{x1} increases the bias of $\hat{\varepsilon}_{x1/x2}$ increases
  - When ρ_{x1x2} decreases the bias of $\hat{\varepsilon}_{x1/x2}$ increases



Summary of Characteristics of $\varepsilon_{x1/x2}$ and $\hat{\varepsilon}_{x1/x2}$

- When $\varepsilon_{x1} = \varepsilon_{x2}$, $\varepsilon_{x1/x2}$ is virtually the same as that of the components; the bias of $\hat{\varepsilon}_{x1/x2}$ increases when V_{x2} increases but ρ_{x1x2} does not have a effect
- If $\varepsilon_{x1}<1.0$ and $\varepsilon_{x2}=1.0$, the covariance matrix of a ratio population tends to have a more homogeneous covariance structure and the bias level of $\hat{\varepsilon}_{x1/x2}$ increases when V_{x2} increases and ρ_{x1x2} decreases

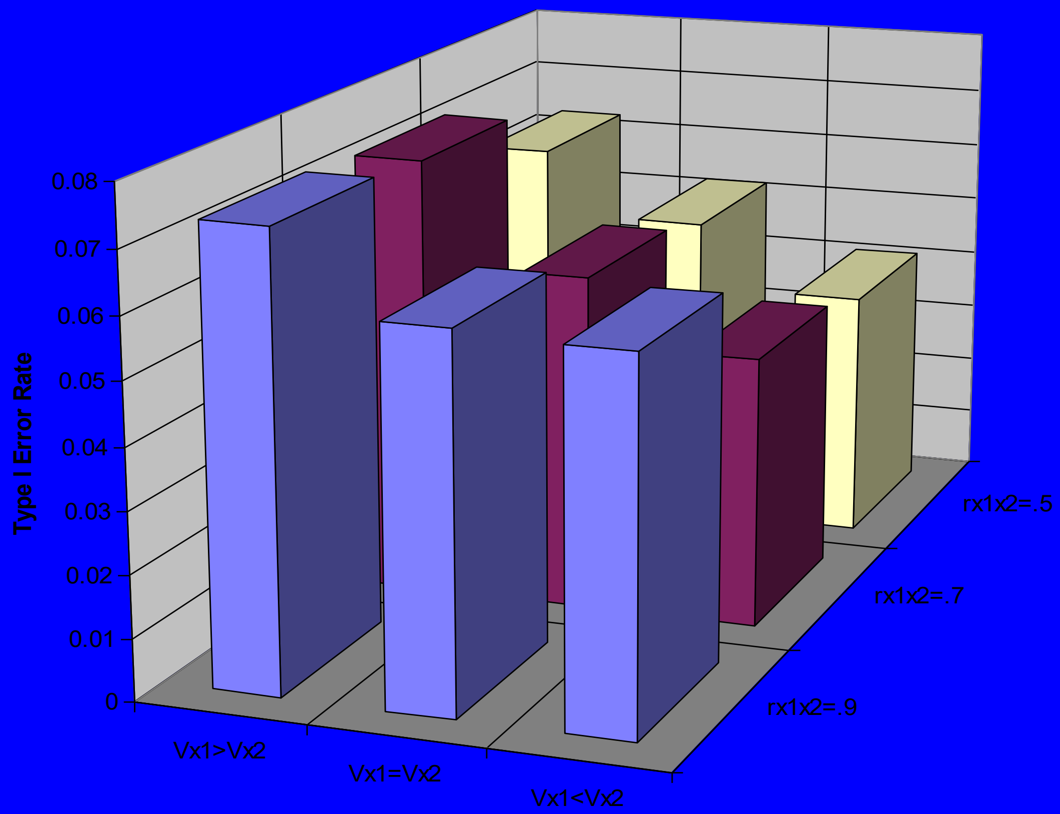


Type I Error Rates

- Effect of $\varepsilon_{x1/x2}$ and ρ_{x1x2} on the type I error rate
  - No substantial deviation of the empirical type I error rate from the nominal level in the condition $\varepsilon_{x1}= \varepsilon_{x2}=1.0$
  - The empirical type I error rate is consistently inflated in the condition $\varepsilon_{x1}= \varepsilon_{x2}=0.7$
  - The empirical type I error rate has a positive relationship with V_{x1}/V_{x2} and ρ_{x1x2} in the condition $\varepsilon_{x1}=0.7$ and $\varepsilon_{x2}=1.0$

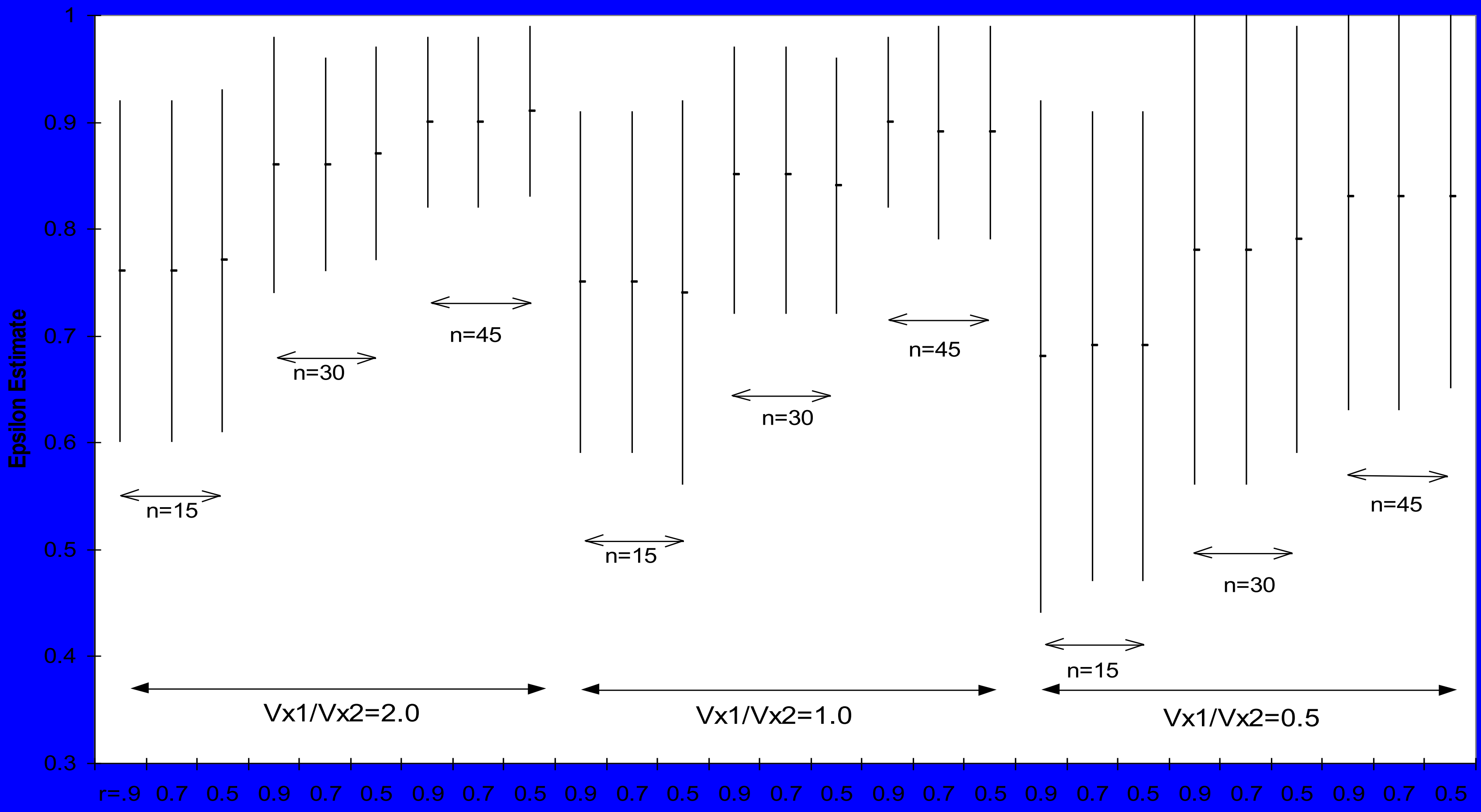
Type I Error Rates (cont.)

- $\varepsilon_{x1}=0.7$, $\varepsilon_{x2}=1.0$



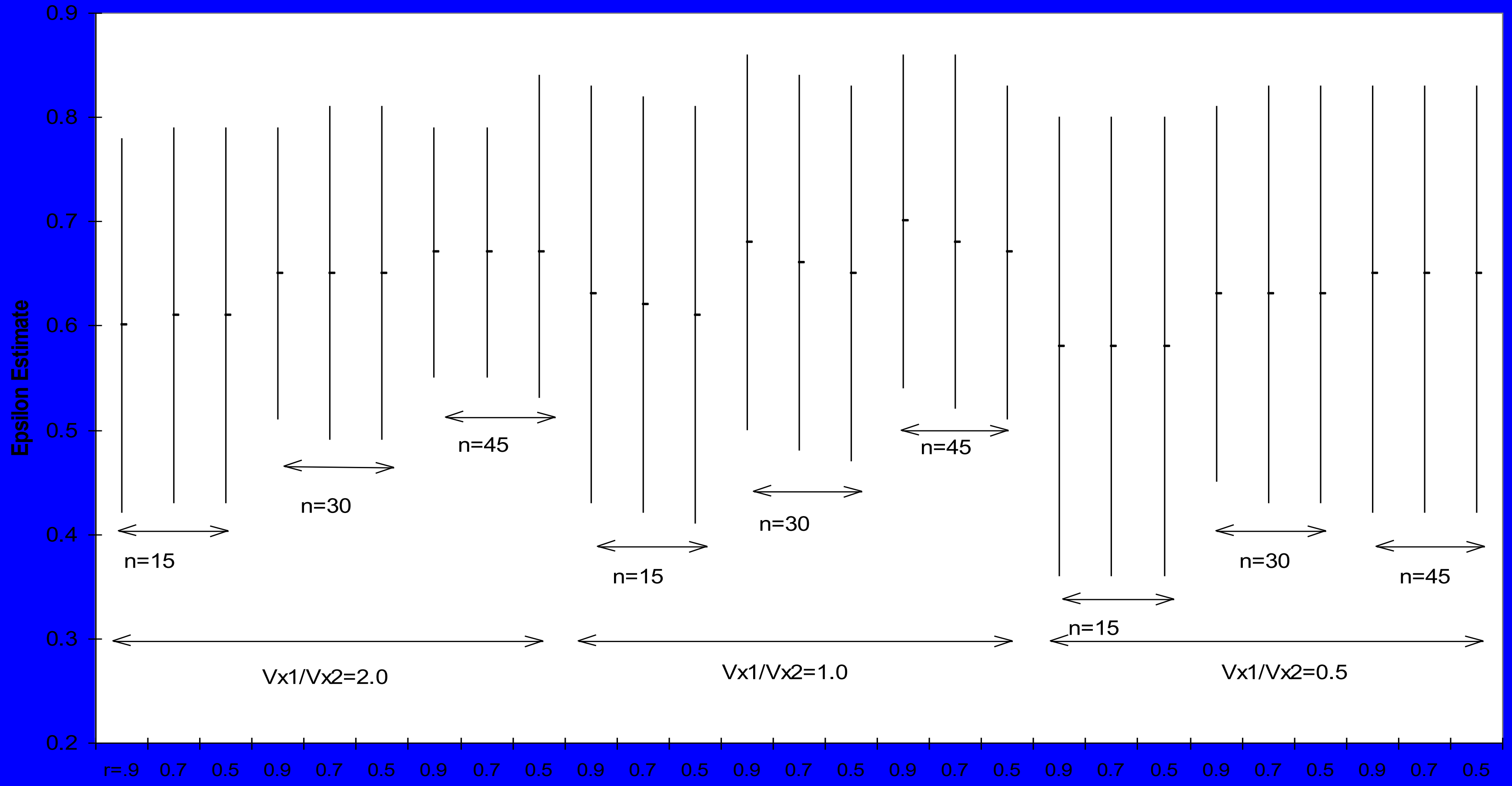
Sampling Estimate $\hat{\varepsilon}_{x1/x2}$

- $\varepsilon_{x1} = \varepsilon_{x2} = 1.0$, $\varepsilon_{x1/x2} = 1.0$



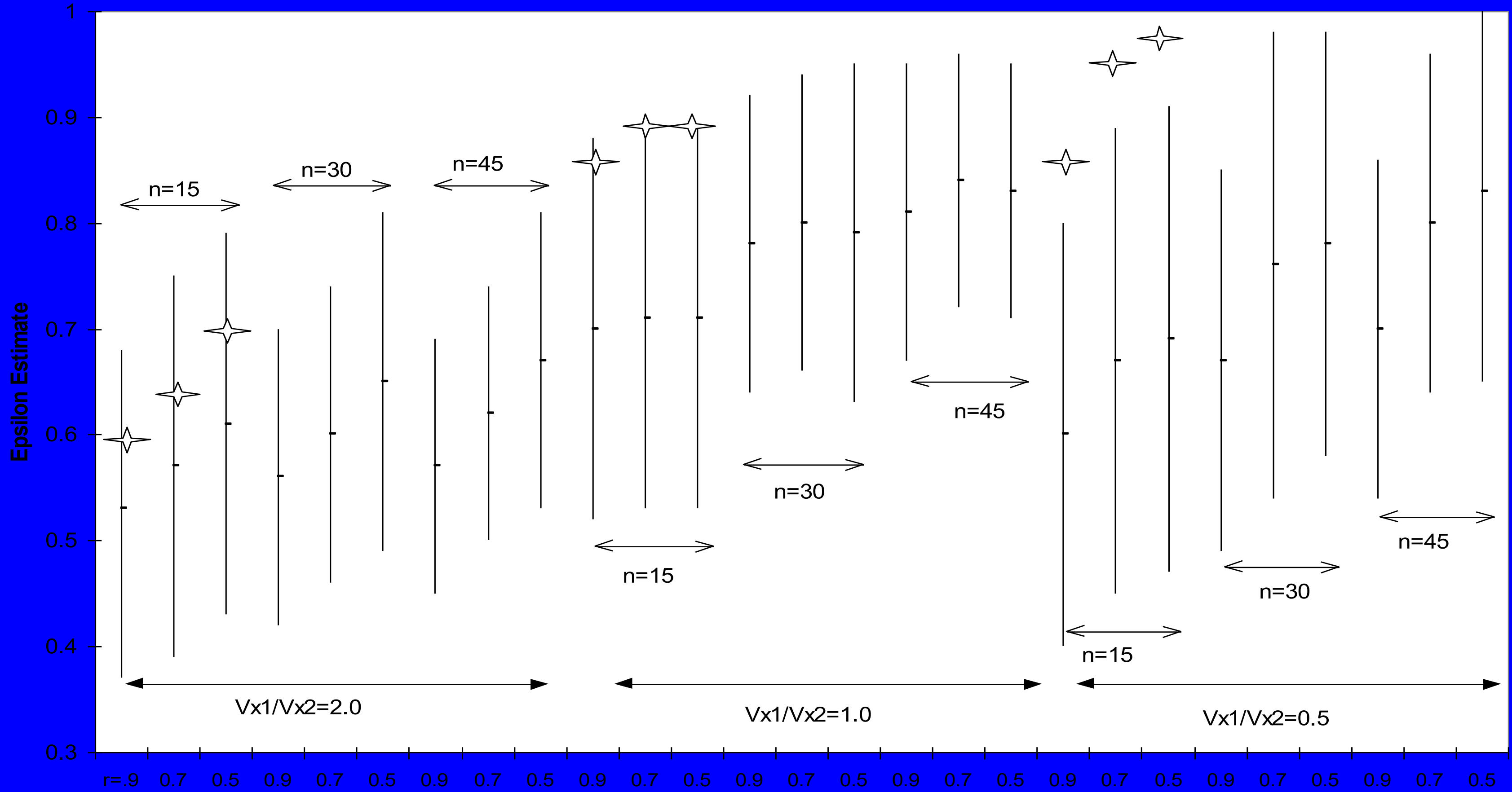
Sampling Estimate $\hat{\varepsilon}_{x1/x2}$ (continue)

- $\varepsilon_{x1} = \varepsilon_{x2} = 0.7$, $\varepsilon_{x1/x2} \approx 0.7$



Sampling Estimate $\hat{}\varepsilon_{x1/x2}$ (continue)

- $\varepsilon_{x1}=0.7$, $\varepsilon_{x2}=1.0$, $\varepsilon_{x1/x2}=$ ✦



Sampling Estimate $\varepsilon_{x1/x2}$ and Type I Error Rates

- The Effect of V_{x1}/V_{x2} on the Type I Error Rates (ρ_{x1x2}=0.7, α=.05, n=15)

ε_{x1}	ε_{x2}	V_{x1}/V_{x2}	$\varepsilon_{x1/x2}$	$\hat{\varepsilon}_{x1/x2}$	Type I Error	$\hat{\varepsilon}_{x1/x2}$ Appropriate Correction[a] $\hat{\varepsilon}$	Appropriate Correction[a] $\tilde{\varepsilon}$	Under-Correction $\hat{\varepsilon}$	Under-Correction $\tilde{\varepsilon}$	Over-Correction $\hat{\varepsilon}$	Over-Correction $\tilde{\varepsilon}$
0.7	0.7	2.0	0.70	0.61(.09)	.061	27.0	25.6	6.0	49.6	67.0	24.8
		1.0	0.71	0.62(.10)	.065	26.4	22.2	8.1	50.0	65.5	27.8
		0.5	0.72	0.58(.11)	.067	16.4	21.9	4.2	34.8	79.4	43.3
0.7	1.0	2.0	0.64	0.57(.09)	.068	32.1	26.1	9.2	50.0	58.7	23.9
		1.0	0.91	0.71(.09)	.054	4.6	4.8	0.3	47.6	95.1	47.6
		0.5	0.96	0.67(.11)	.055	1.3	17.1	0.1	15.9	98.6	67.0

[a] Appropriate correction refers to percent of times an adjustment in the F test would be "appropriate" defined as $|\hat{\varepsilon}_{x1/x2}-\varepsilon_{x1/x2}|<=0.05$. Under-correction is defined as insufficient adjustment which occurs when $\hat{\varepsilon}_{x1/x2}>\varepsilon_{x1/x2}+0.05$, and over-correction is defined as too conservative adjustment which occurs when $\hat{\varepsilon}_{x1/x2}<\varepsilon_{x1/x2}-0.05$.

A Summary of the Effect of Vx1/Vx2 on the Type I Error Rates (cont.)

- The over-adjustment of the ^ε-adjusted F test is more pronounced in the two conditions $\varepsilon_{x1} = \varepsilon_{x2} = 1.0$, and $\varepsilon_{x1} = 0.7$ and $\varepsilon_{x2} = 1.0$
- A large bias and extreme variability of ^ε-adjusted F tests would be expected when $V_{x1} < V_{x2}$, especially with small sample size
- The H-F correction reduces the likelihood of an over-adjustment, but it does not increase the probability of the correct correction
- The results suggest that if the denominator has lower variation than the numerator and sample size is large, the risk of the over-correction in RM ANOVA tests could be substantially reduced

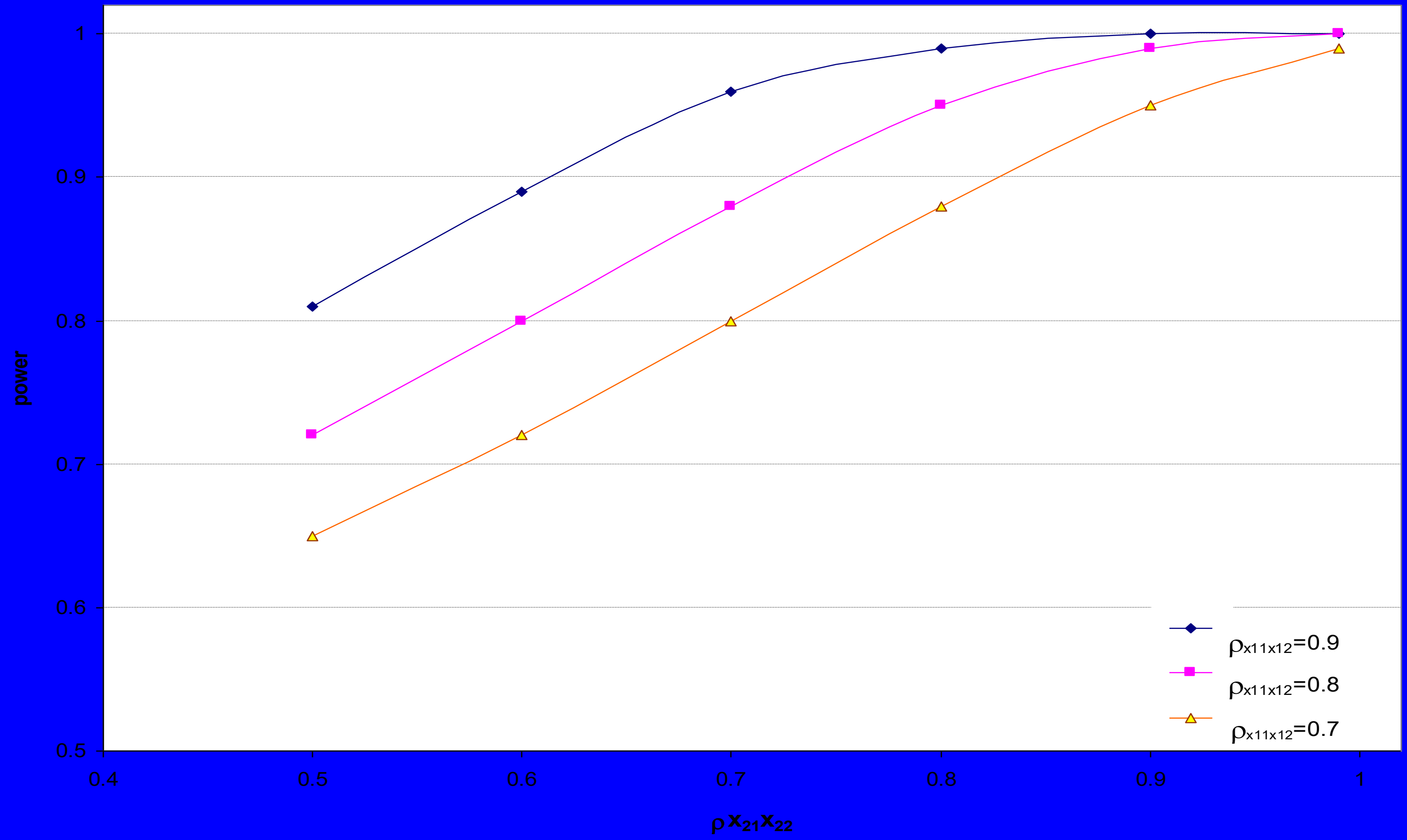
Effect of Using Ratio Variables on Power

- **The Effect of the Between trial Correlation of the Numerator on Power**

	ρ_{x11x12}=0.5	0.6	0.7	0.8	0.9
X_1 (numerator)					
λ	8.44	10.55	**14.06**	21.09	42.19
Cohen's d	1.06	1.19	**1.37**	1.68	2.37
Power	0.78	0.87	**0.95**	0.99	≈1.00
Y (ratio)					
ρ_{y1y2}	0.28	0.45	**0.62**	0.79	0.96
λ	10.43	13.66	**19.78**	35.81	189.11
Cohen's d	1.18	1.35	**1.62**	2.19	5.02
Power	0.86	0.94	**0.99**	0.999	≈1.00

Effect of Using Ratio Variables on Power (cont.)

- The Effect of the Between Trial Correlation of the Denominator on Power



Summary of the Power Investigation

- **Given other factors constant, power is affected by not only $\rho_{x1i,x1j}$ but also $\rho_{x2i,x2j}$**
  - **When $\rho_{x1i,x1j}$ increases, power increases**
  - **When $\rho_{x2i,x2j}$ increases, power increases**
- **The results indicate that if the denominator varies over trials, measuring the denominator once could result in higher power and and overestimate of the treatment effect**

Implication of the Investigation

- **Different models should be used to derive an appropriate deflation model in an empirical study**
- **When a ratio variable is used the reliability should be examined according to the ratio variable data**
- **If homogeneity of the denominator variable and large sample size are present, it may reduce likelihood of bias in $\hat{\varepsilon}_{x1/x2}$ and protect the type I error rate**
- **Given the mean difference and variance are held constant, power has a positive relationship with both ρ_{x11x12} and ρ_{x21x22}**